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TECHNICAL NOTES

Non-Darcy mixed convection boundary layer flow on a vertical cylinder in a saturated porous medium

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INTRODUCTION

THE STUDY of flow through porous media finds applications in a number of fields such as in geothermal reservoirs, packed bed reactors, heat storage beds, and beds of fossil fuels. Bejan [1] has given an excellent review of the free convection flows in porous media. Free convection problems involving different body shapes have been studied recently using the non-Darcy model [2–7]. However, the mixed convection flow in porous media using the non-Darcy model has been studied only for a horizontal flat plate [8].

The aim of this study is to consider the non-Darcy mixed convection flow over a thin vertical cylinder embedded in a saturated porous medium. Both forced flow and buoyancy force dominated cases have been considered. The governing partial differential equations have been solved numerically using the Keller box method. The results are presented for a buoyancy parameter which covers the entire regime of mixed convection flow ranging from pure forced convection to pure free convection.

GOVERNING EQUATIONS

We consider an isothermal vertical cylinder of radius r_0 and wall temperature T_w immersed in a saturated porous medium having temperature T_∞ and free stream velocity u_∞. We assume that the properties of the fluid and the porous medium are everywhere isotropic and homogeneous and the convective fluid and the porous medium are everywhere in local thermodynamic equilibrium. Viscous dissipation has been neglected. We also invoke the Boussinesq approximation. Under the foregoing assumptions, the boundary layer equations with the non-Darcy flow model governing the flow are given by [6]

$$(ru)_x + (rv)_r = 0 \tag{1}$$

$$u_r + (K^*/\nu)(u^2)_r = \mu^{-1}\rho_\infty g\beta K T_r \tag{2}$$

$$uT_x + vT_r = (\alpha/r)(rT_r)_r. \tag{3}$$

The appropriate boundary conditions are

$$\left.\begin{array}{lll} v = 0, & T = T_w & \text{on} \quad r = r_0 \\ u = u_\infty, & T = T_\infty & \text{as} \quad r \to \infty \end{array}\right\} \quad \text{for} \quad x \geq 0. \tag{4}$$

As in the classical case of mixed convection flow over a cylinder [9], we consider both forced flow and buoyancy force dominated cases.

Forced flow dominated case

For this case, we apply the following transformations:

$$\eta = [(r^2 - r_0^2)/2r_0](u_\infty/\alpha x)^{1/2}, \quad \xi = (2/r_0)(\alpha x/u_\infty)^{1/2}$$
$$u = r^{-1}\psi_r, \quad v = -r^{-1}\psi_x, \quad (r/r_0)^2 = 1 + \xi\eta \tag{5a}$$

$$\psi = r_0(\alpha u_\infty x)^{1/2} f(\xi,\eta), \quad u = u_\infty f'$$
$$v = -(r_0/r)(\alpha u_\infty/x)^{1/2}(f + \xi f_\xi - \eta f')/2 \tag{5b}$$

$$\lambda = Gr/Re, \quad Gr = g\beta K(T_w - T_\infty)r_0/\nu^2, \quad \nu = \mu/\rho_\infty$$
$$Re = u_\infty r_0/\nu, \quad A = u_\infty K^*/\nu, \quad (T - T_\infty)/(T_w - T_\infty) = G(\xi,\eta) \tag{5c}$$

to equations (1)–(3), and we find that equation (1) is satisfied identically and equations (2) and (3) reduce to

$$(1 + 2Af')f'' = B\lambda G' \tag{6}$$

$$(1 + \xi\eta)G'' + (2^{-1}f + \xi)G' = (\xi/2)(f'G_\xi - G'f_\xi). \tag{7}$$

Boundary conditions (4) reduce to

$$f = G - 1 = 0 \quad \text{at} \quad \eta = 0;$$
$$f' - 1 = G = 0 \quad \text{as} \quad \eta \to \infty, \ \xi \geq 0. \tag{8}$$

It may be remarked that equations (6) and (7) for $\xi = A = 0$, $B = \pm 1$ reduce to those of Darcian mixed convection flow over a vertical flat plate studied by Cheng [10]. The constant $B = 1$ for aiding flow and $B = -1$ for opposing flow.

The skin friction and heat transfer can be expressed as

$$C_f(Re_x/Pr)^{1/2} = -f''(\xi,0)$$
$$Nu(Re_x Pr)^{-1/2} = -G'(\xi,0) \tag{9}$$

where

$$C_f = 2\tau_w/\rho u_\infty^2, \quad \tau_w = -\mu(u_r)_w$$
$$Nu = xq_w/[k(T_w - T_\infty)], \quad q_w = -k(T_r)_w, \quad Re_x = u_\infty x/\nu. \tag{10}$$

Here the parameter A measures the relative importance of the inertial effect. The buoyancy parameter $\lambda > 0$ for aiding flow and $\lambda < 0$ for opposing flow. Equation (6) gives the relation between skin friction and heat transfer parameters.

We have compared the local heat flux along a vertical cylinder with that of a vertical flat plate embedded in the same porous medium. The ratio of the heat transfer rate is given by

$$q_{cyl}/q_{fp} = [-G'(\xi,0)]_{cyl}/[-G'(0)]_{fp}. \tag{11}$$

Similarly, the ratio of the total surface heat transfer can be expressed as

$$Q_{cyl}/Q_{fp} = \xi_L^{-1}\int_0^{\xi_L} [-G'(\xi,0)]_{cyl}/-G'(0)]_{fp}\,d\xi \tag{12a}$$

$$\xi_L = (2/r_0)(\alpha L/u_\infty)^{1/2}. \tag{12b}$$

Buoyancy force dominated case

In order to derive the appropriate equations for this case, we define the following new variables:

$$\eta_1 = x^{-1}(Ra_x)^{1/2}(r_0/2)[(r/r_0)^2 - 1]$$
$$\xi_1 = 2(x/r_0)(Ra_x)^{-1/2}, \quad \psi = \alpha r_0(Ra_x)^{1/2}F(\xi_1,\eta_1)$$
$$u = r^{-1}\psi_r, \quad v = -r^{-1}\psi_x \tag{13a}$$

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NOMENCLATURE

A, A_1	non-Darcy parameters
C_f	skin friction coefficient
f, F	transformed stream functions
g	acceleration due to gravity
G, H	dimensionless temperatures
Gr, k	Grashof number and thermal conductivity
K, K^*	permeability of the medium and inertial coefficients
L, Nu, Pr	cylinder length, Nusselt number and Prandtl number
q, Q	local and total heat transfer rates, respectively
r, x	radial and axial coordinates
r_0, Ra_x	radius of cylinder and modified local Rayleigh number
Re, Re_x	Reynolds numbers defined with respect to r_0 and x
T	temperature
u, v	velocity components in the x- and r-directions.

Greek symbols

α, β	thermal diffusivity and coefficient of thermal expansion
η, η_1, λ	pseudo-similarity variables and buoyancy parameter
μ, ν, ρ	viscosity, kinematic viscosity and density
ξ, ξ_1, τ	stretched streamwise coordinates and shear stress
ψ	stream functions.

Superscript

$'$	derivatives with respect to η and η_1.

Subscripts

cyl, fp	cylinder and flat plate, respectively
r, x	derivatives with respect to r and x, respectively
w, ∞	conditions at the wall and in the free stream, respectively.

$$Ra_x = \rho_\infty g\beta Kx(T_w - T_\infty)/\mu\alpha, \quad (r/r_0)^2 = 1+\xi_1\eta_1$$

$$u = \mu^{-1}\rho_\infty g\beta K(T_w - T_\infty)F'(\xi_1, \eta_1) \tag{13b}$$

$$v = -(\alpha/r\xi_1)[F+\xi_1 F_{\xi_1} - \eta_1 F']$$

$$T - T_w = (T_w - T_\infty)H(\xi_1, \eta_1), \quad \lambda_1 = Re/Gr$$

$$A_1 = g\beta KK^*(T_w - T_\infty)\nu^2. \tag{13c}$$

In terms of these variables, equation (1) is identically satisfied and equations (2) and (3) reduce to

$$(1+2A_1F')F'' = B_1H' \tag{14}$$

$$(1+\xi_1\eta_1)H''+(2^{-1}F+\xi_1)H' = (\xi_1/2)(F'H_{\xi_1} - H'F_{\xi_1}). \tag{15}$$

The boundary conditions are

$$F = H-1 = 0 \quad \text{at} \quad \eta_1 = 0;$$

$$F' - \lambda_1 = H = 0 \quad \text{as} \quad \eta \to \infty, \ \xi_1 \geqslant 0. \tag{16}$$

It may be noted that equations (14)–(16) are the same as those of Kumari *et al.* [4] if we put $B_1 = 1$ and $\lambda_1 = 0$. Also equations (14)–(16) reduce to those of the flat plate case studied by Plumb and Huenfeld [2] when we put $B_1 = 1$, $A_1 = \lambda_1 = 0$. Furthermore, equations (14)–(16) reduce to those of Darcian flow studied by Cheng [10] and Kumari *et al.* [6] if we put $A_1 = \lambda_1 = 0$, $B_1 = 1$. Also $B_1 = 1$ for aiding flow and $B_1 = -1$ for opposing flow.

The Nusselt number and skin friction coefficient for this case are given by

$$Nu(Re\,Pr)^{-1/2} = -\lambda_1^{-1/2}H'(\xi_1, 0) \tag{17a}$$

$$C_f Re = (\lambda_1\xi_1)^{-1}F''(\xi_1, 0). \tag{17b}$$

The expressions λ_1, A_1, ξ_1, η_1, $H'(\xi_1, 0)$ and $F''(\xi_1, 0)$ are related to the corresponding expressions of the forced flow dominated case at

$$\lambda_1 = \lambda^{-1}, \quad A_1 = A\lambda, \quad \xi_1 = \lambda^{-1/2}\xi, \quad \eta_1 = \lambda^{1/2}\eta$$

$$H'(\xi_1, 0) = \theta'(\xi, 0)\lambda^{-1/2}, \quad F''(\xi_1, 0) = \lambda^{-2}f''(\xi, 0). \tag{18}$$

The local and total heat transfer ratios can be expressed as

$$q_{cyl}/q_{fp} = [-H'(\xi_1, 0)]_{cyl}/[-H'(0)]_{fp} \tag{19a}$$

$$Q_{cyl}/Q_{fp} = \xi_{1L}^{-1}\int_0^{\xi_{1L}} \{[-H'(\xi_1, 0)]_{cyl}/[-H'(0)]_{fp}\}\,d\xi_1 \tag{19b}$$

$$\xi_{1L} = 2(L/r_0)(Ra_L)^{-1/2}, \quad Ra_L = \rho_\infty g\beta KL(T_w - T_\infty)/\mu\alpha. \tag{20}$$

RESULTS AND DISCUSSION

We have carried out the computations for various values of λ ranging from pure forced convection to pure free convection ($0 \leqslant \lambda \leqslant \infty$). For $0 \leqslant \lambda \leqslant 20$, equations (6) and (7) under conditions (8) were used and for $1 \leqslant \lambda \leqslant \infty$ ($1 \geqslant \lambda_1 \geqslant 0$), equations (14) and (15) with conditions (16) were used. As in ref. [9], the two sets of equations gave the same results. The governing equations have been solved numerically using the Keller box method which is described in detail in ref. [11].

In order to assess the accuracy of our method, we have compared our heat transfer ($-G'(0)$) at $\xi = A = 0$ with that of Cheng [10] and $\xi_1 = \lambda_1 = B_1 - 1 = 0$ with those of Plumb and Huenfeld [2]. Also the heat transfer ratio (q_{cyl}/q_{fp}) obtained from equations (14) and (15) for $B_1 - 1 = \lambda_1 = 0$ has been compared with that of Kumari *et al.* [6]. They all agree at least up to three decimal places. The comparison is not shown here for the sake of brevity.

The effects of the buoyancy parameter λ and the curvature parameter ξ on the heat transfer and skin friction ($-G'(\xi, 0)$, $-f''(\xi, 0)$) for both aiding flow ($\lambda > 0$) and opposing flow ($\lambda < 0$) are given in Fig. 1. It is observed that both heat transfer and skin friction increase with λ ($\lambda > 0$) or ξ. For $\lambda > 0$, they decrease with λ. The reason for such a behaviour is that both momentum and thermal boundary layer thicknesses reduce as λ ($\lambda > 0$) or ξ increases.

Figure 2 presents the heat transfer ($-G'(\xi, 0)$) for the buoyancy parameter λ which covers the entire regime of mixed convection ranging from pure forced convection to pure free convection. Forced and free convection asymptotes are also shown in this figure. It is observed that the heat transfer tends to its forced convection value as $\lambda \to 0$ and to its free convection value as $\lambda \to \infty$. The curve for $-G'(\xi, 0)$ lies above the asymptotes and the maximum difference occurs near $\lambda = 10$. This difference reduces as the non-Darcian parameter A increases. Following the 5% deviation rule suggested by Sparrow *et al.* [12] for the subdivisions of forced, mixed and free convection flows, we can classify the flows as follows.

For non-Darcy flow ($A = 1$)

$$0 < \lambda < 0.6 \quad \text{forced convection} \tag{21a}$$

$$0.6 < \lambda < 20 \quad \text{mixed convection} \tag{21b}$$

$$20 < \lambda \quad \text{free convection.} \tag{21c}$$

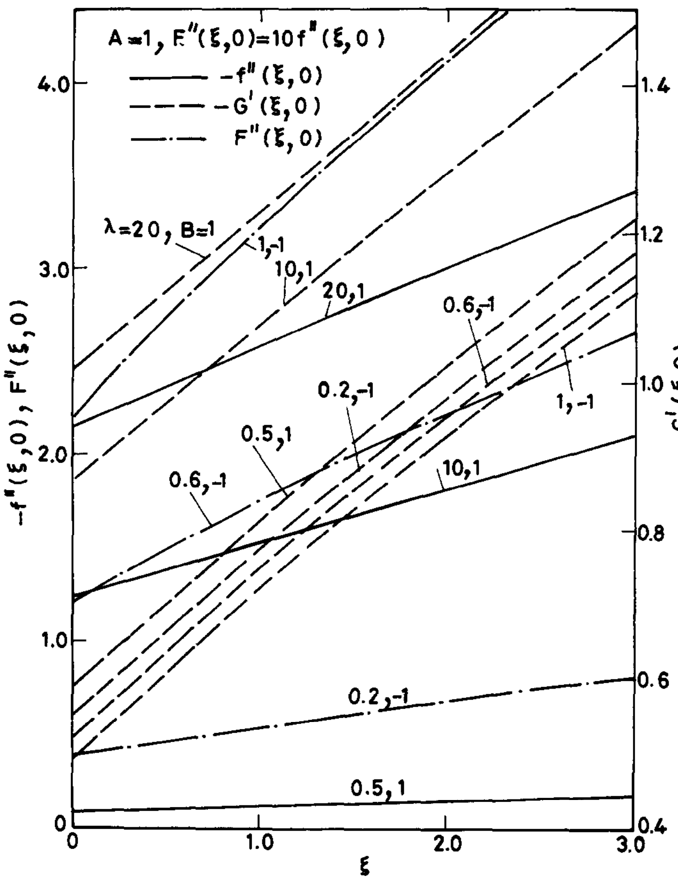

FIG. 1. Effects of the buoyancy parameter λ and the curvature parameter ξ on the skin friction and heat transfer.

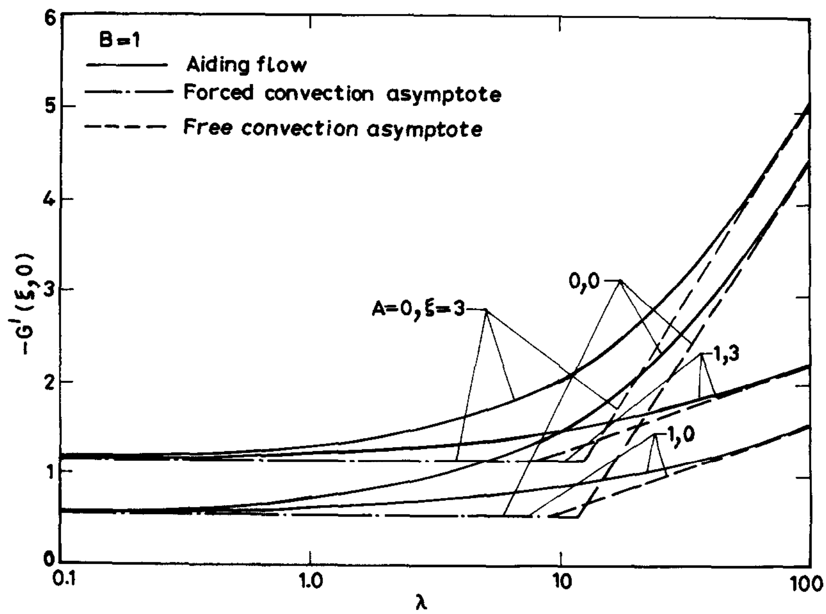

FIG. 2. Variation of heat transfer with the buoyancy parameter and non-Darcy parameter A.

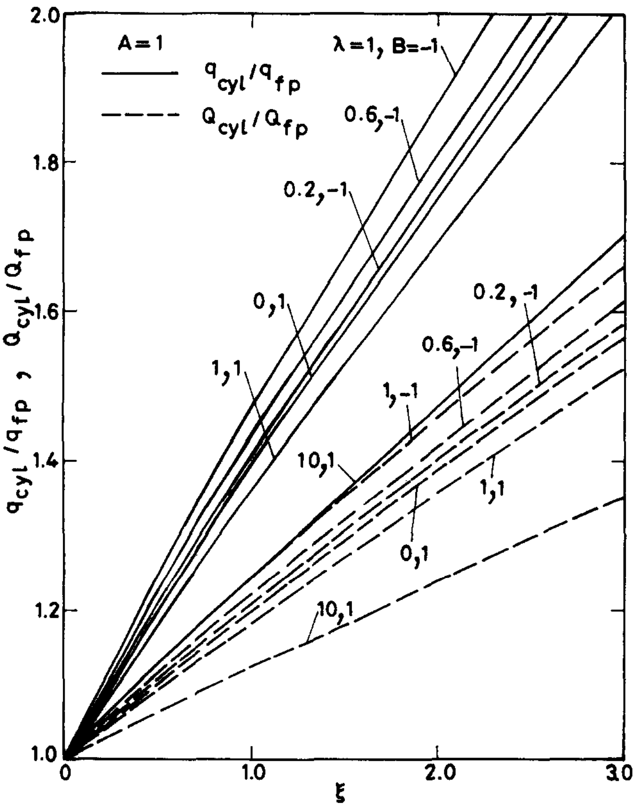

FIG. 3. Effect of the buoyancy parameter λ on the local and total heat transfer ratios.

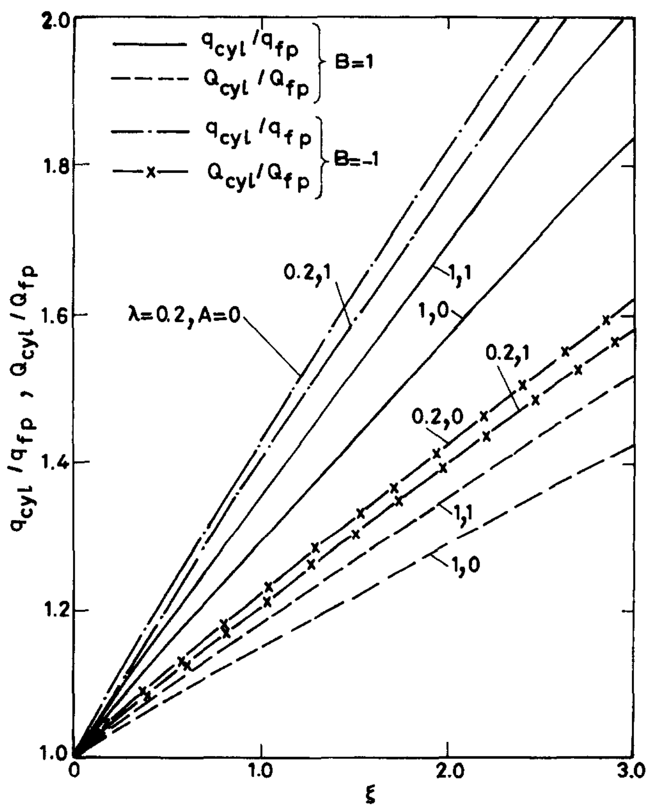

FIG. 4. Effect of the non-Darcy parameter A on the local and total heat transfer ratios.

For Darcy flow ($A = 0$)

$0 < \lambda < 0.3$	forced convection	(22a)
$0.3 < \lambda < 35$	mixed convection	(22b)
$35 < \lambda$	free convection.	(22c)

Figures 3 and 4 present the ratios of the local and the total surface heat transfer of the cylinder and that of the flat plate (q_{cyl}/q_{fp}, Q_{cyl}/Q_{fp}) for different values of buoyancy, curvature and non-Darcy parameters (λ, ξ, A). Both the local and the total heat transfer ratios (q_{cyl}/q_{fp}, Q_{cyl}/Q_{fp}) increase with ξ for any given λ. For aiding flow ($\lambda > 0$), q_{cyl}/q_{fp} and Q_{cyl}/Q_{fp} decrease as λ ($\lambda > 0$) increases, but they increase as A increases. However, the opposite trend is observed for the case of opposing flow.

CONCLUSIONS

The skin friction and heat transfer parameters are found to increase with increasing buoyancy force for aiding flow and the opposite trend is observed for opposing flow. The results tend to their forced or free convection values when the buoyancy parameter tends to zero or infinity. The results based on the non-Darcy flow model differ considerably from those obtained by using the classical Darcy flow model. Both the heat transfer and skin friction increase with the curvature parameter.

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On the heat transfer from a cylindrical fin

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1. INTRODUCTION

HEAT TRANSFER from a fin involves two mechanisms, namely conduction within the fin and convection from the fin surface to the surrounding fluid. The boundary condition at the fin–fluid interface is the continuity of heat flux and temperature. Thus the conduction and convection problems are coupled. Conventionally, however, the heat transfer characteristics of fins are determined from the given literature value of the heat transfer coefficient that is assumed constant over the entire fin surface. In this conventional approach, henceforth called the simple model, the actual fin–fluid interaction is not taken care of. The heat transfer coefficient actually varies along the fin surface, and depends upon the fin conductance as well as the Reynolds and Prandtl numbers. The correct method, henceforth called the complete model, is to solve the conduction problem for the fin and the convection problem for the fluid simultaneously.

For rectangular fins complete model studies have been carried out by Sparrow and Chyu [1] for forced convection, by Sparrow and Acharya [2] for natural convection, and by Sunden [3, 4] for mixed convection but only for a fixed Prandtl number of 0.7. A very simple method, based on a similar solution approach, was reported in ref. [5] for a wide range of Prandtl numbers. The effect of Prandtl number on the heat transfer from a rectangular fin has also been studied by Sunden [6]. For cylindrical fins complete model studies have been carried out in refs. [7, 8], and in ref. [9] for forced, natural and mixed convective flow, respectively, but for a Prandtl number of 0.7 only. Moreover, the analysis of Huang and Chen [7] has some deficiencies, as detailed in ref. [9].

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